KINEMATIC COUPLINGS

The good, the bad, the ugly….

Defining constraint

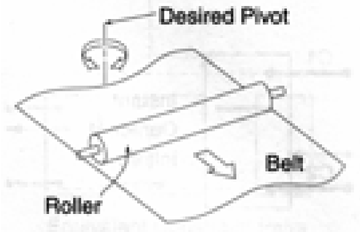


FIGURE 1.9.1

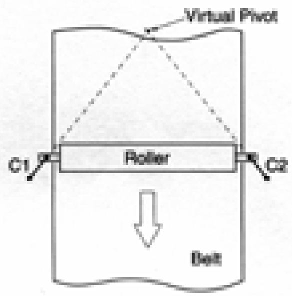


FIGURE 1.9.2

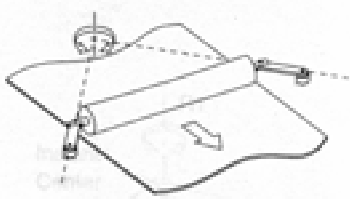


FIGURE 1.9.3

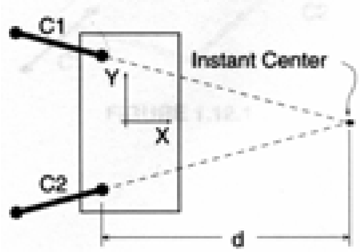


FIGURE 1.10.1

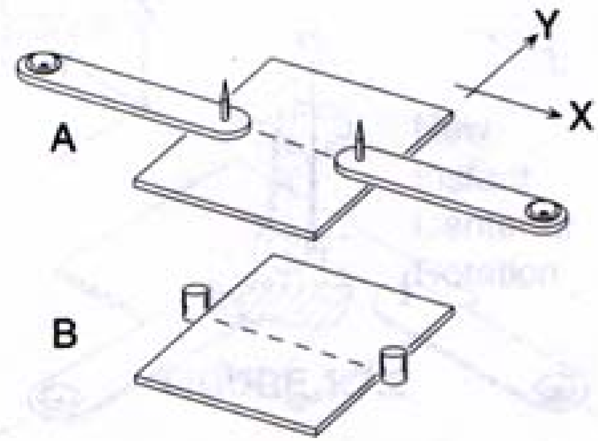


FIGURE 1.5.1

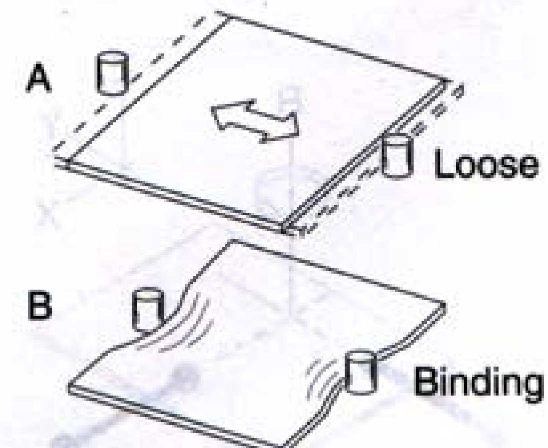


FIGURE 1.5.2

Clever use of constraint

Penalties for over constraint

Exact constraint (Kinematic) design

Exact Constraint: Number of constraint points = DOF to be constrained

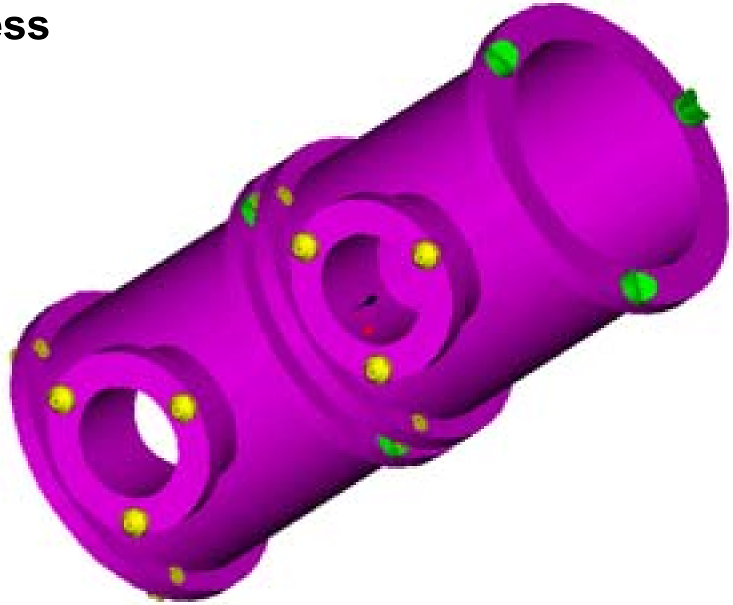
These constraints must be independent!!!

Assuming couplings have rigid bodies, equations can be written to describe movements

Design is deterministic, saves design and development $

KCs provide repeatability on the order of parts' surface finish

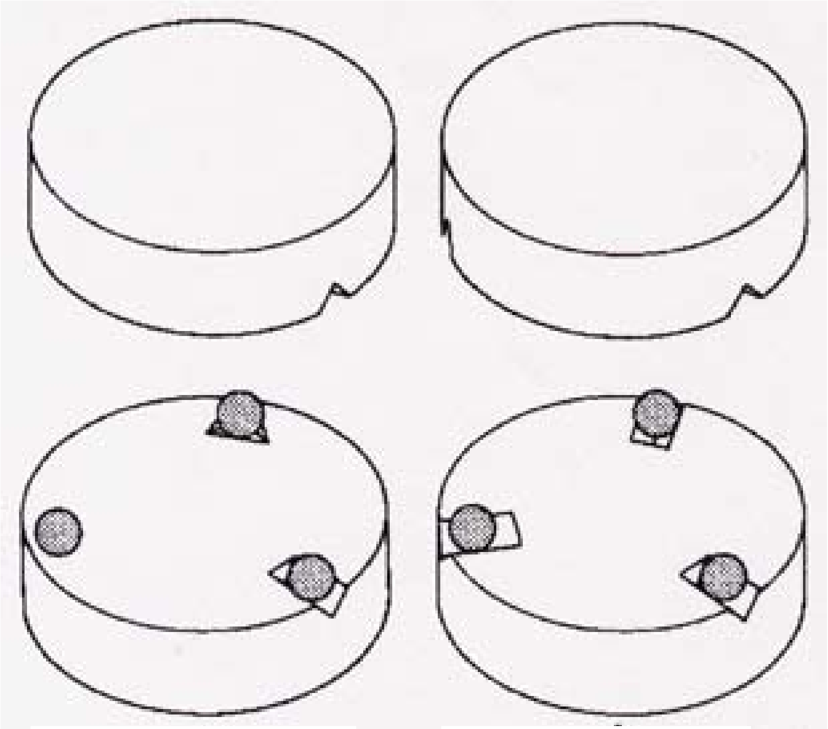
- ¼ micron repeatability is common
- Managing contact stresses are the key to success



Making life easier

“Kinematic Design”, “Exact Constraint Design”…..the issues are:

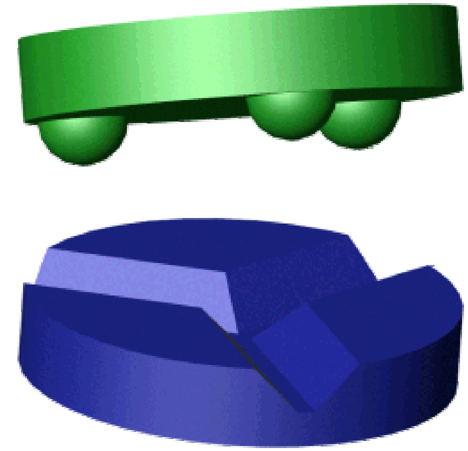
- KNOW what is happening in the system
- Manage forces and deflections
- Minimize stored energy in the coupling
- Know when “Kinematic Design” should be used
- Know when “Elastic Averaging” should be used (next week)



Kelvin Clamp Boyes Clamp

Picture from Precision Machine Design, Slocum, A. H.

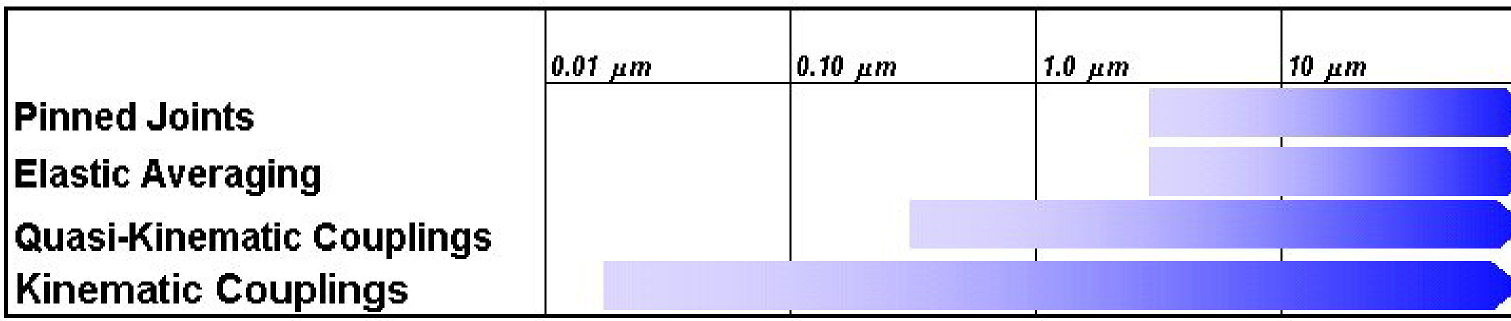
Kinematic couplings



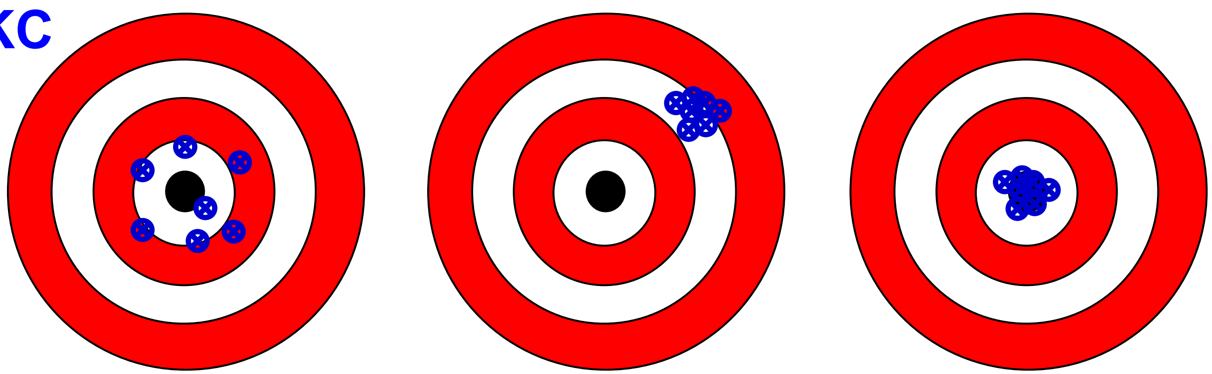
Kinematic Couplings:

- Deterministic Coupling
- # POC = # DOF
- Do Not Allow Sealing Contact
- Excellent Repeatability

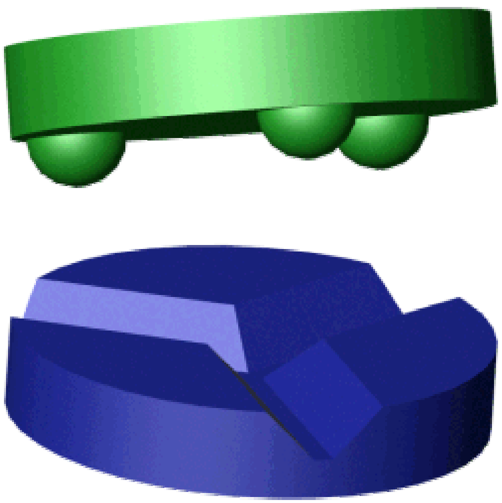
Performance

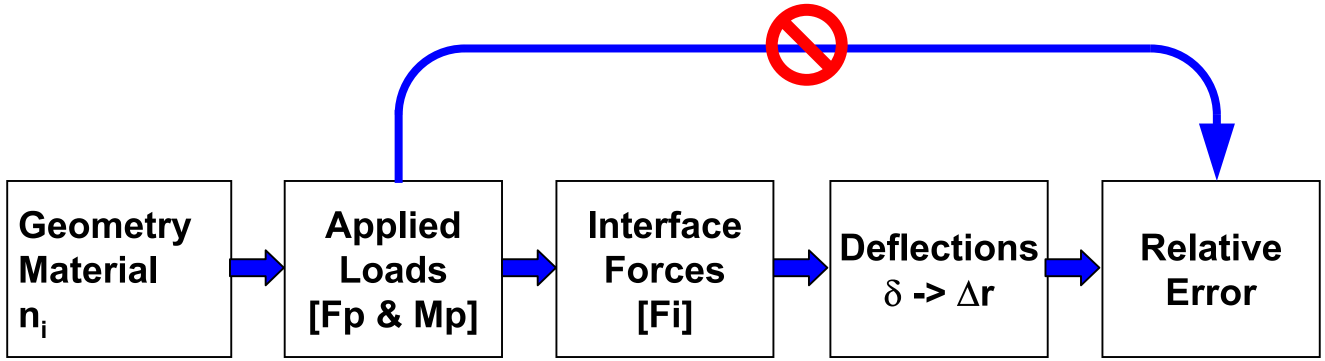


Power of the KC



Modeling Kinematic coupling error motions



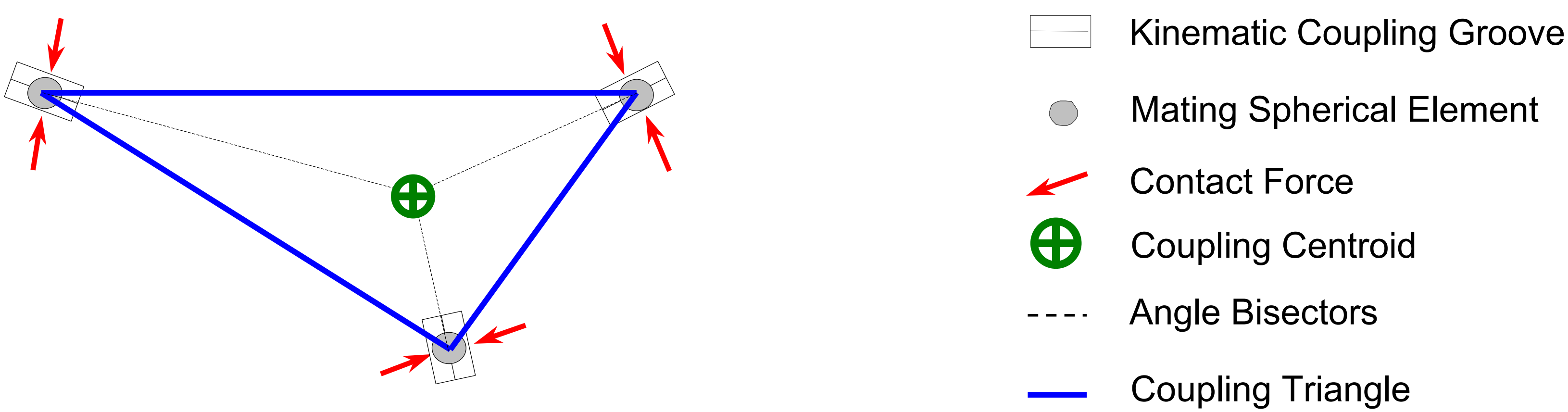


6 Unknown Forces & 6 Equilibrium Equations

$\Sigma F_i = F_P \qquad \Sigma M_i = M_P$

Hertzian Point Contact for Local Displacements

$\delta_i = f(E_B, E_G, \nu_B, \nu_G, R_B, R_G)$



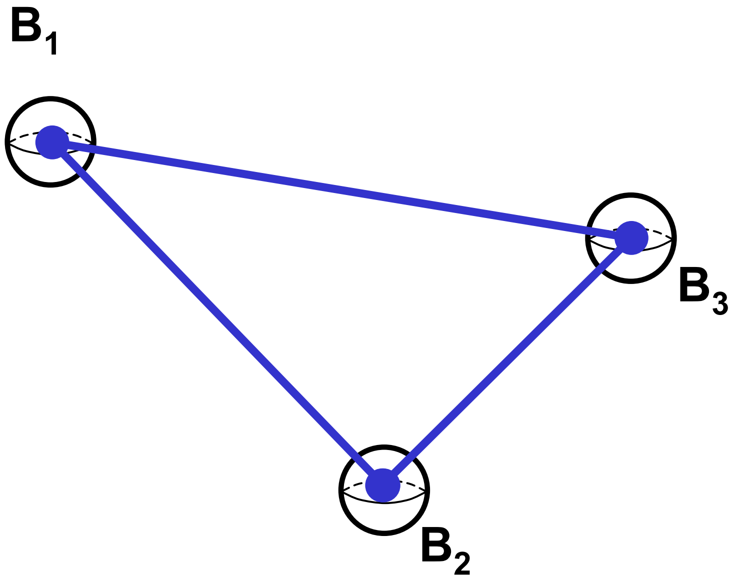
KC error motion analysis

Need δ_x, δ_y, δ_z, ε_x, ε_y , ε_z to predict effect of non-repeatability

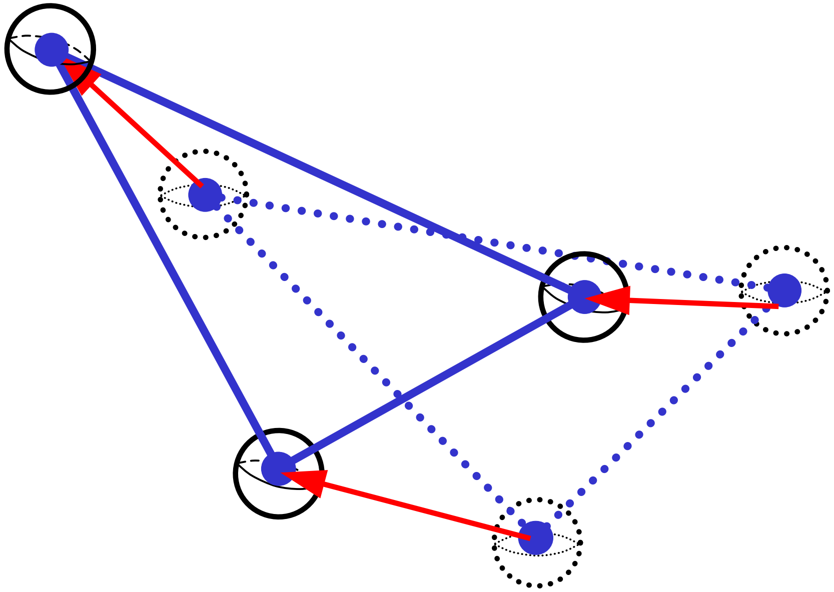
Hertz deflections -> displacements of ball centers

Three ball centers form a plane

Analyze relative position of "before" and "after" planes for error motions



Original Positions

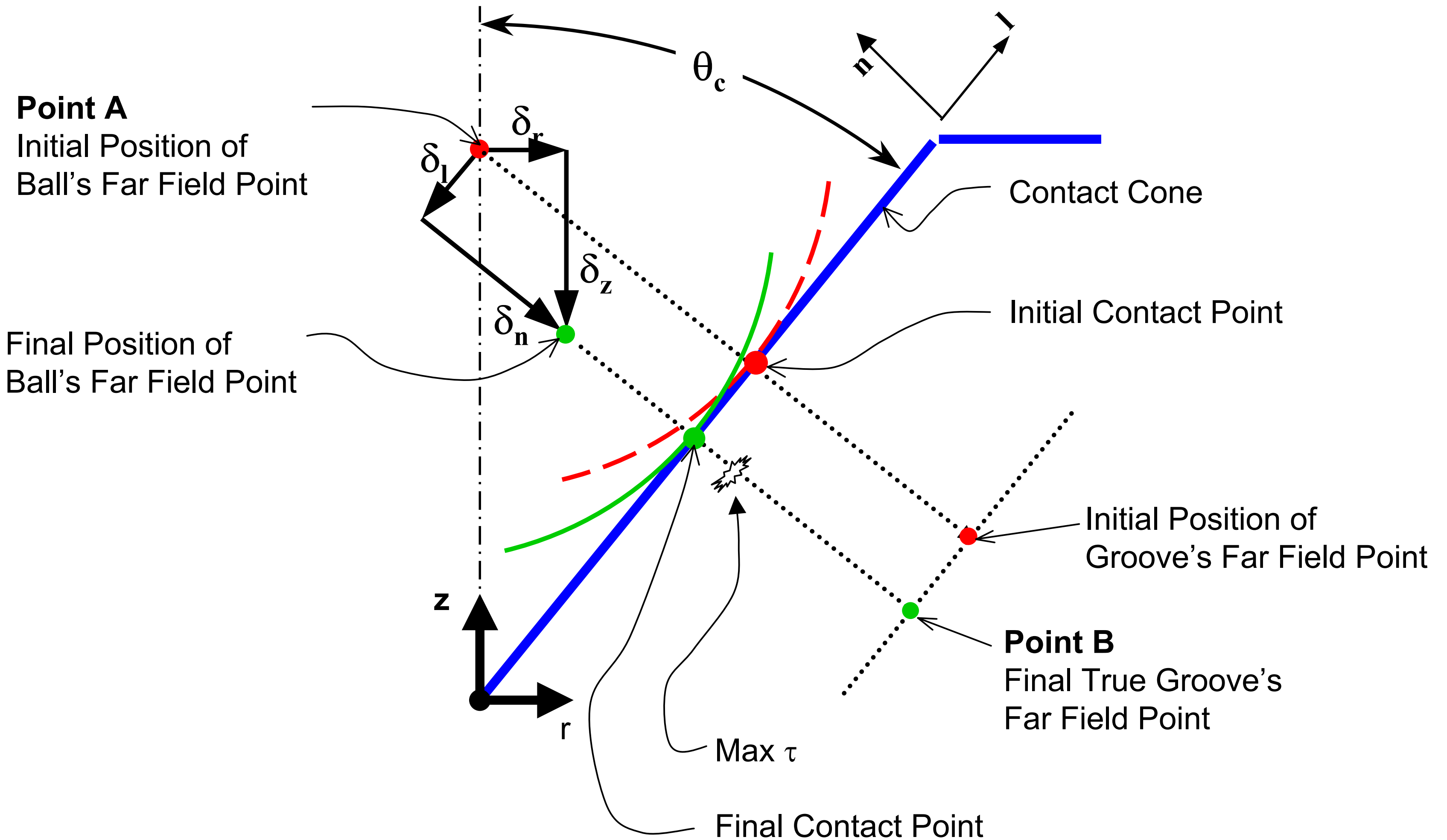


Final Positions

Kinematic couplings and distance of approach

How do we characterize motions of the ball centers?

δ_n = distance of approach



Max shear stress occurs below surface, in the member with larges R

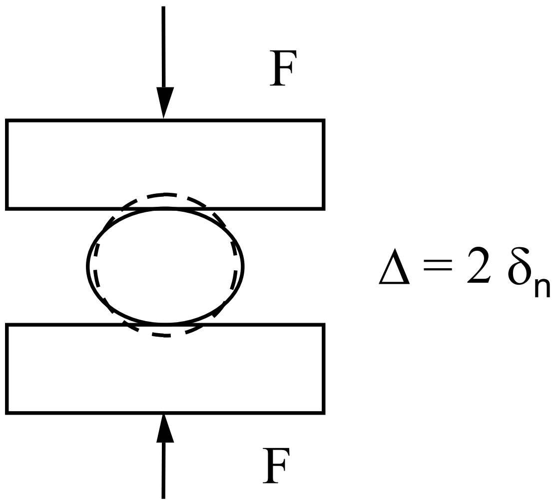
Contact mechanics – Hertzian contact

Heinrich Hertz – 1st analytic solution for "near" point contact

KC contacts are modeled as Hertz Contacts

Enables us to determine stress and distance of approach, δ_n

Radii	Ronemaj	**1.00E+06**
	Ronemin	**0.06250**
	Rtwomaj	**0.25000**
	Rtwomin	**-0.06500**
Load	Applied load F	**13**
	Phi (degrees)	**0**
	Max contact stress	**10,000**
Modulus	Elastic modulus Eone	**3.00E+07**
	Elastic modulus Etwo	**4.40E+05**
ν Ratio	Poisson's ratio vone	**0.3**
	Poisson's ratio vtwo	**0.3**
	Equivalent modulus Ee	4.77E+05
	Equivalent radius Re	0.2167
Stress	Contact pressure	12,162
	Stress ratio (must be less than 1)	1.22
	Deflection at the one contact interface	
Deflection	Deflection (µunits)	829



Key Hertzian physical relations

Equivalent radius and modulus:

$$R_e = \frac{1}{\frac{1}{R_{1major}} + \frac{1}{R_{1minor}} + \frac{1}{R_{2major}} + \frac{1}{R_{2minor}}} \qquad E_e = \frac{1}{\frac{1 - \;^{2}_{1}}{E_1} + \frac{1 - \;^{2}_{2}}{E_2}}$$

cos(θ) function (ϕ is the angle between the planes of principal curvature of the two bodies)

$$\cos \;\; = R_e\left[\left(\frac{1}{R_{1major}} - \frac{1}{R_{1minor}}\right)^2 + \left(\frac{1}{R_{2major}} - \frac{1}{R_{2minor}}\right)^2 + 2\left(\frac{1}{R_{1major}} - \frac{1}{R_{1minor}}\right)\left(\frac{1}{R_{2major}} - \frac{1}{R_{2minor}}\right)\cos 2 \;\;\right]^{1/2}$$

Solution to elliptic integrals estimated with curve fits

$$= 1.939\ e^{-5.26} + 1.78\ e^{-1.09} + 0.723/ \;\; + 0.221$$

$$= 35.228\ e^{-0.98} - 32.424\ e^{-1.0475} + 1.486 \;\; - 2.634$$

$$= -0.214\ e^{-4.95} - 0.179 \;\;^{2} + 0.555 \;\; + 0.319$$

Contact Pressure	Distance of Approach	Major Contact Axis	Minor Contact Axis
$q = \frac{3F}{2\ \ cd} \quad 1.5$ tensile for metals	$= \left(\frac{2F^2}{3R_eE_e^2}\right)^{1/3}$	$c = \alpha \left(\frac{3FR_e}{2E_e}\right)^{1/3}$	$\beta \left(\frac{3FR_e}{2E_e}\right)^{1/3}$

KEY Hertzian relation scaling laws

Contact Pressure is proportional to:

- Force to the 1/3rd power
- Radius to the –2/3rd power
- Modulus to the 2/3rd power

Distance of approach is proportional to:

- Force to the 2/3rd power
- Radius to the –1/3rd power
- Modulus to the –2/3rd power

Contact ellipse diameter is proportional to:

- Force to the 1/3rd power
- Radius to the 1/3rd power
- Modulus to the –1/3rd power

DO NOT ALLOW THE CONTACT ELLIPSE TO BE WITHIN ONE DIAMETER OF THE EDGE OF A SURFACE!

Calculating error motions in kinematic couplings

Motion of ball centers -> Centroid motion in 6 DOF -> Δx, Δy, Δz at X, Y, Z

- **Coupling Centroid Translation Errors**

$$c = \frac{1}{L_{1c}} \qquad \frac{2}{L_{2c}} \qquad \frac{3}{L_{3c}} \qquad \frac{L_{1c} \qquad L_{2c} \qquad L_{3c}}{3}$$

- **Rotations**

$$x = \frac{z1}{L_1 \ 23} \cos \quad 23 \qquad \frac{z2}{L_2 \ 31} \cos \quad 31 \qquad \frac{z3}{L_3 \ 12} \cos \quad 12$$

$$y = \frac{z1}{L_1 \ 23} \sin \quad 23 \qquad \frac{z2}{L_2 \ 31} \sin \quad 31 \qquad \frac{z3}{L_3 \ 12} \sin \quad 12$$

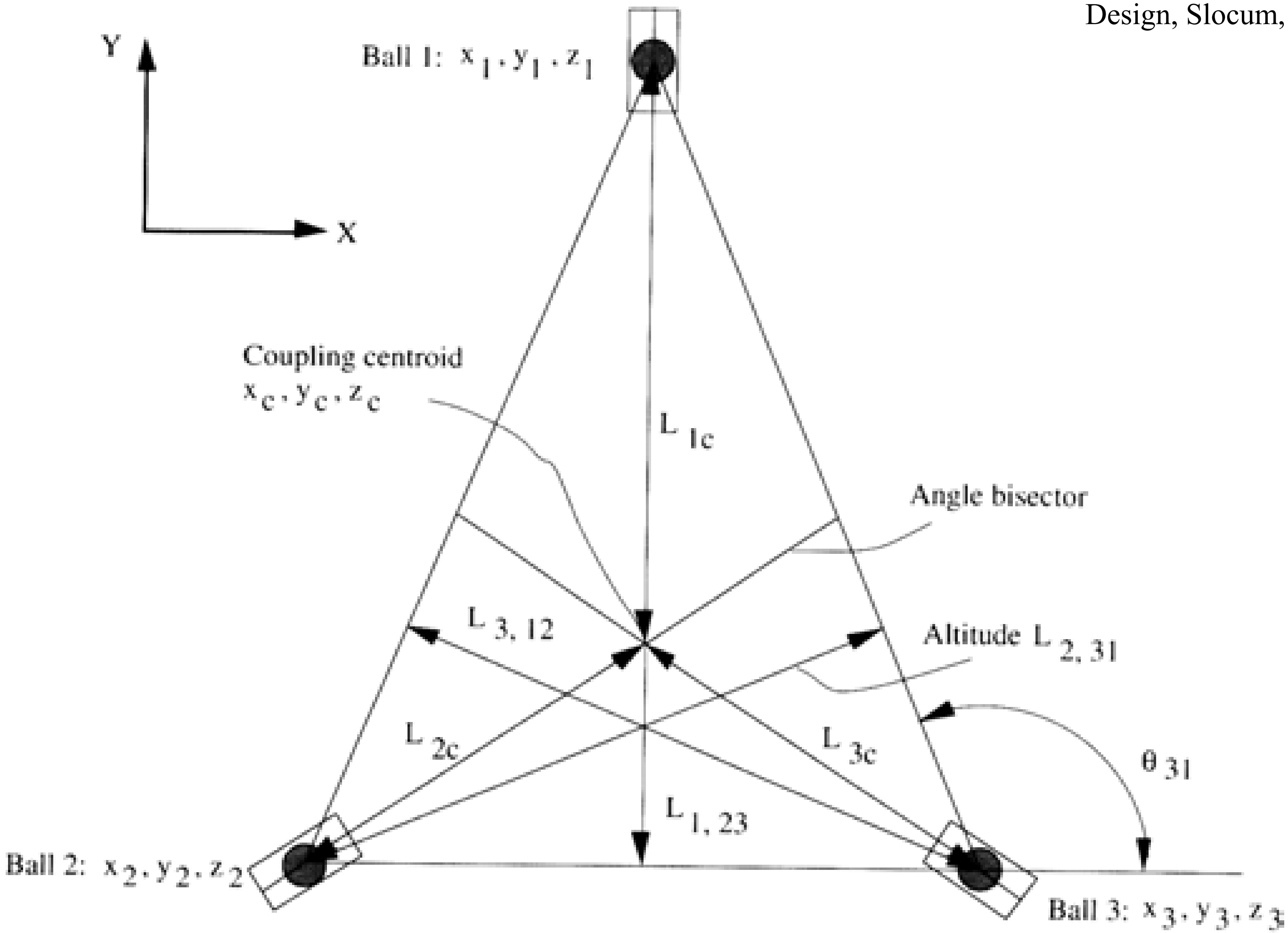
$$z1 = \frac{\sqrt{\quad B1 \quad 1 \qquad B2 \quad 2 \quad ^2 \qquad B1 \quad 1 \qquad B2 \quad 2 \quad ^2}}{\sqrt{\quad x_1 \quad x_c \quad ^2 \qquad y_1 \quad y_c \quad ^2}} \ \mathrm{SIGN} \quad B1 \quad 1 \qquad B2 \quad 2 \qquad \longrightarrow \qquad z = \frac{z1 \qquad z2 \qquad z3}{3}$$

- **Error At X, Y, Z (includes translation and sine errors)**

$$\begin{array}{c} x \\ y \\ z \\ 1 \end{array} = \begin{array}{cccc} 1 & z & y & x \\ z & 1 & x & y \\ y & x & 1 & z \\ 0 & 0 & 0 & 1 \end{array} \quad \begin{array}{c} X \quad x_c \\ Y \quad y_c \\ Z \quad z_c \\ 1 \end{array} \quad \begin{array}{c} X \quad x_c \\ Y \quad y_c \\ Z \quad z_c \\ 1 \end{array}$$

Kinematic coupling centroid displacement

Picture from Precision Machine Design, Slocum, A. H.



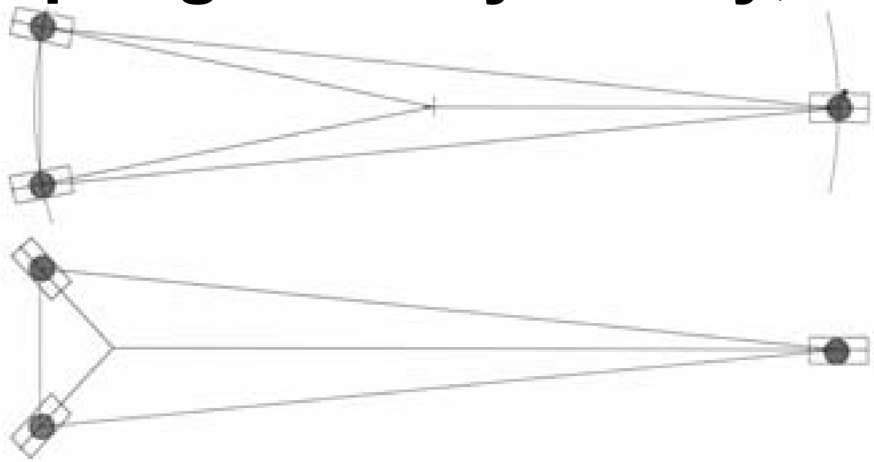
General design guidelines

1. Location of the coupling plane is important to avoid sine errors
2. For good stability, normals to planes containing contact for vectors should bisect angles of coupling triangle
3. Coupling triangle centroid lies at center circle that coincides with the three ball centers
4. Coupling centroid is at intersection of angle bisectors
5. These are only coincident for equilateral triangles
6. Mounting the balls at different radii makes crash-proof
7. Non-symmetric grooves make coupling idiot-proof

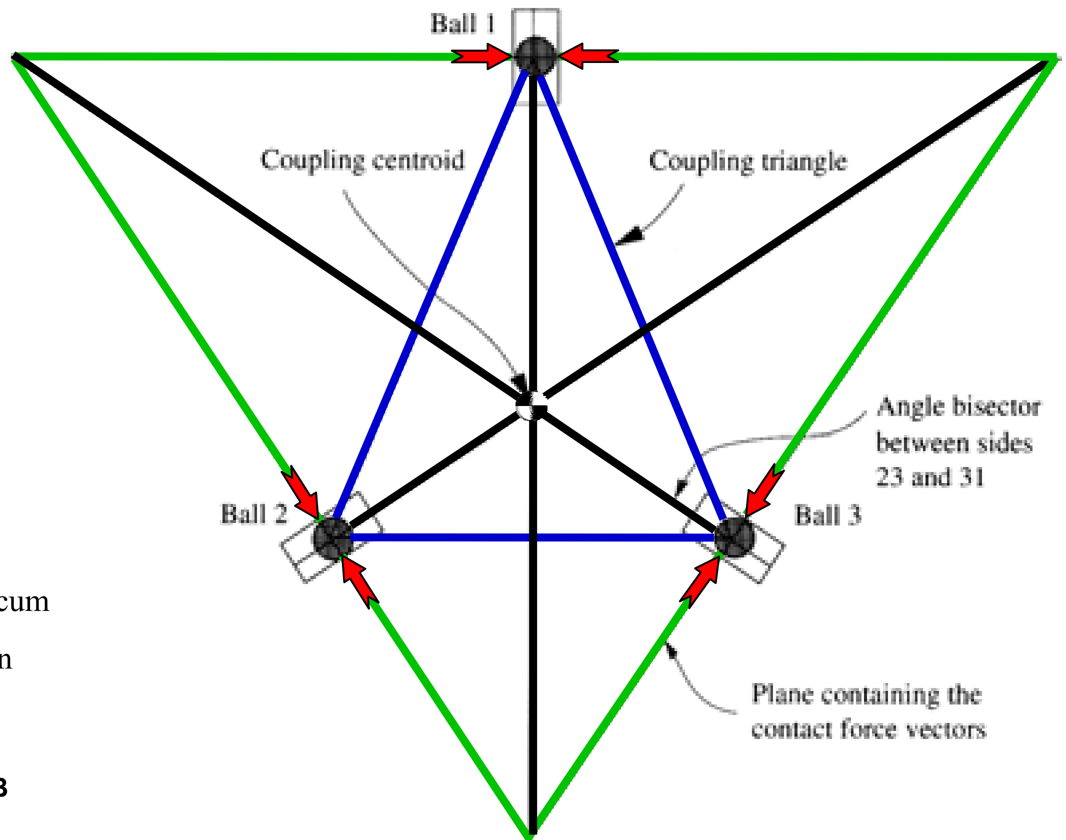
Kinematic coupling stability theory, instant centers

Poor Design

Good Design



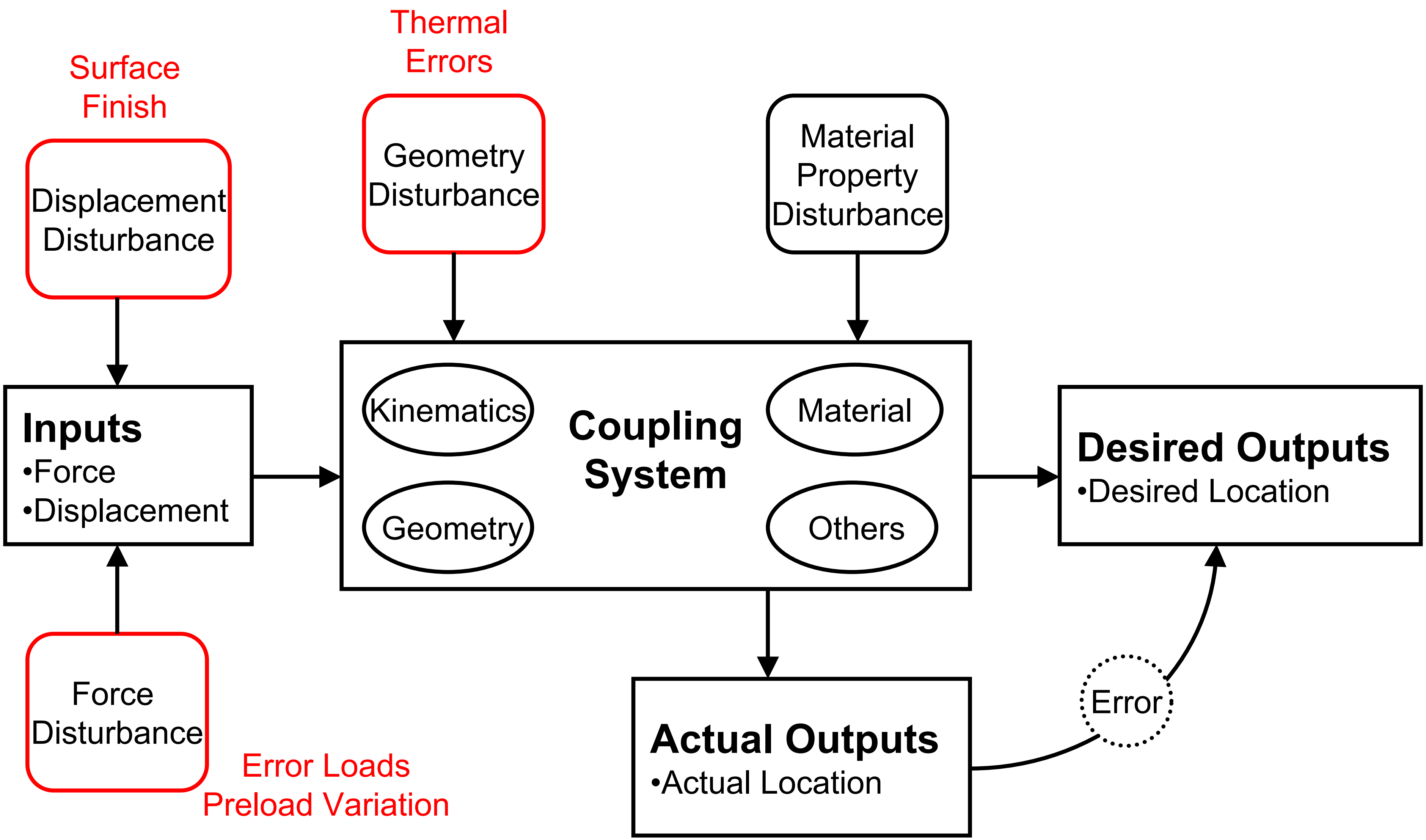
Picture from Precision Machine Design, Slocum, A. H.



*Pictures courtesy Alex Slocum

Precision Machine Design

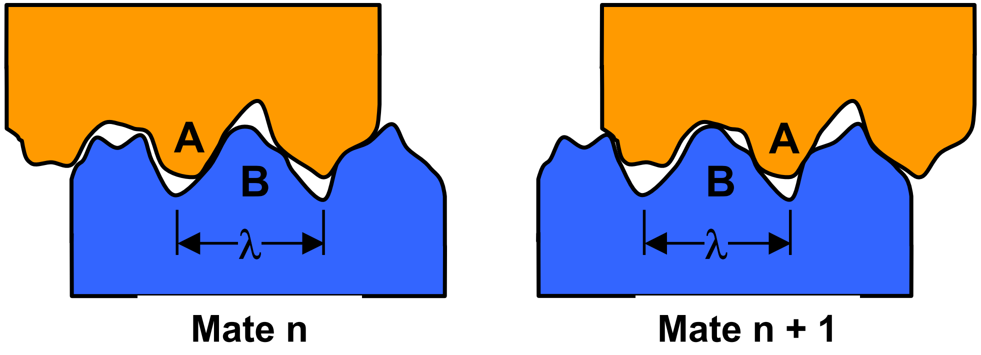
Sources of errors in kinematic couplings



Problems with physical contact (and solutions)

Surface topology (finish):

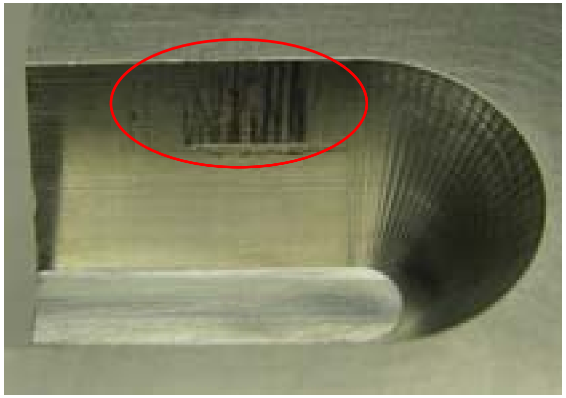
- 50 cycle repeatability ~ 1/3 μm Ra
- Friction depends on surface finish!
- Finish should be a design spec
- Surface may be brinelled if possible



Wear and Fretting:

- High stress + sliding = wear
- Metallic surfaces = fretting
- Use ceramics if possible (low μ and high strength)
- Dissimilar metals avoids "snowballing"

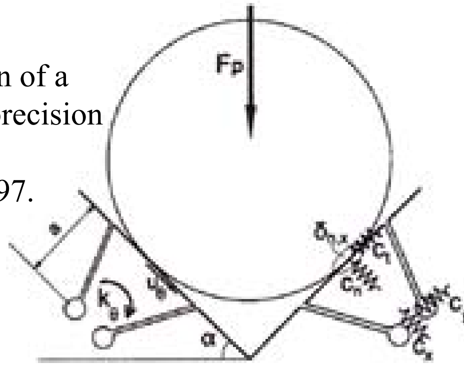
Wear on Groove



Friction:

- Friction = Hysteresis, stored energy, over constraint
- Flexures can help (see right)
- Lubrication (high pressure grease) helps
  - Beware settling time and particles
- Tapping can help if you have the "magic touch"

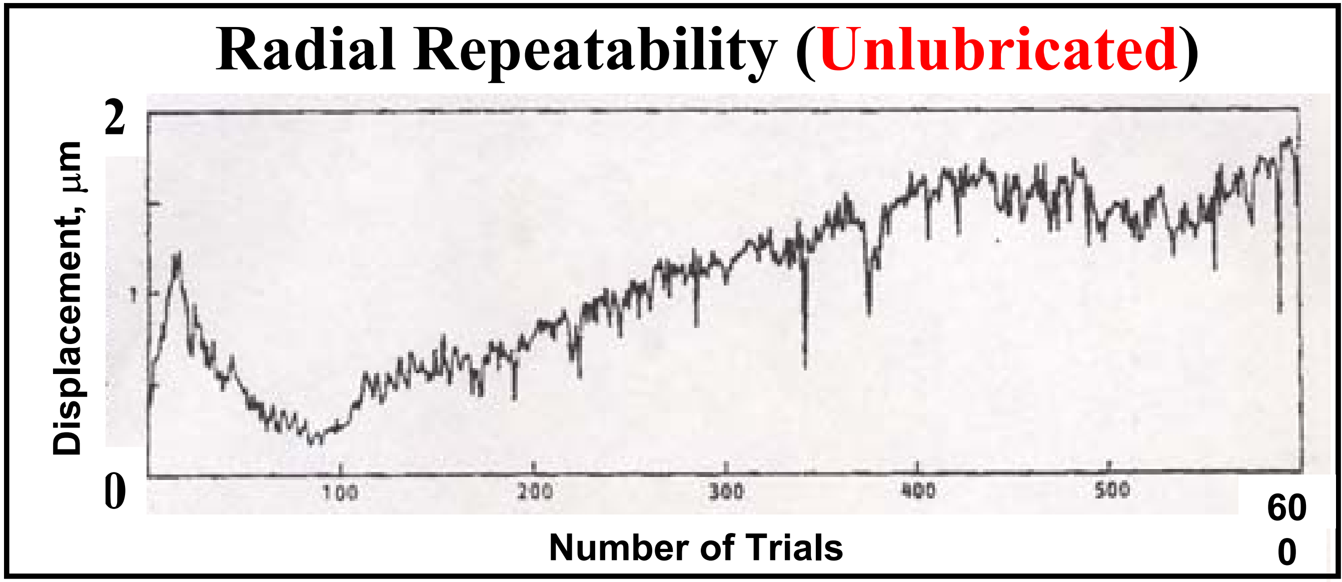
Picture from:
Schouten, et. al., "Design of a kinematic coupling for precision applications", Precision Engineering, vol. 20, 1997.

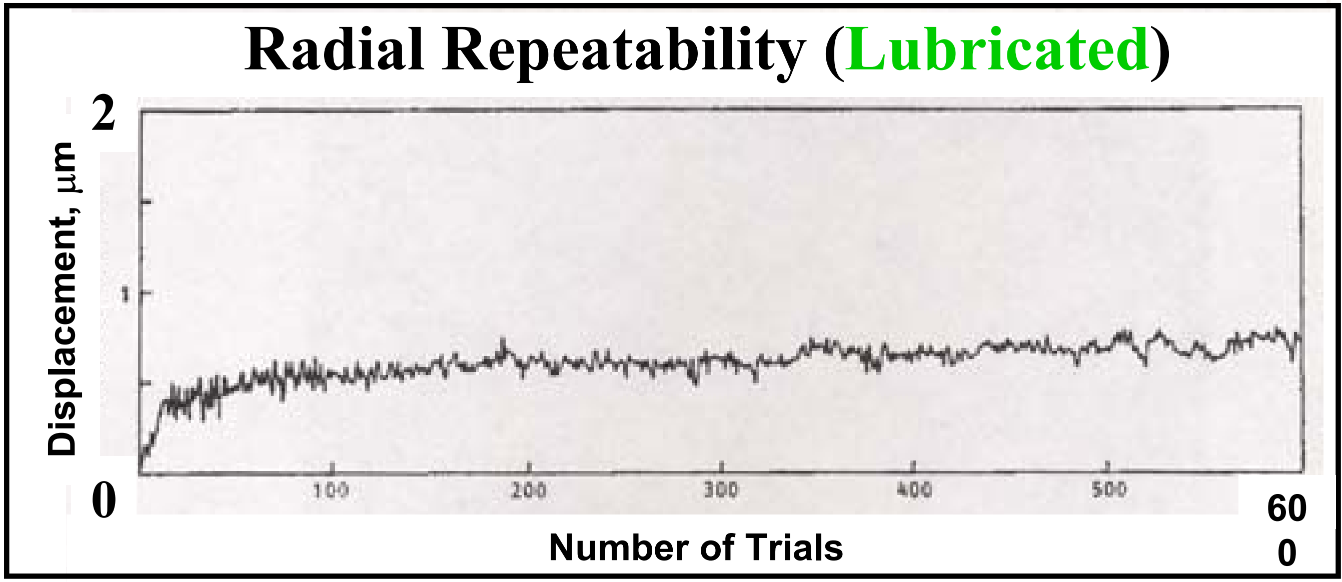


Ball in V-Groove with Elastic Hinges

Experimental results – Repeatability & lubrication

Slocum, A. H., Precision Engineering, 1988: Kinematic couplings for precision fixturing – Experimental determination of repeatability and stiffness





Practical design of kinematic couplings

Design

- Specify surface finish or brinell on contacting surfaces
- Normal to contact forces bisect angles of coupling triangle!!!

Manufacturing & Performance

- Repeatability = f (friction, surface, error loads, preload variation, stiffness)
- Accuracy = f (assembly) unless using and ARKC

Precision Balls (ubiquitous, easy to buy)

- Baltec sells hardened, polished kinematic coupling balls or…..

Grooves (more difficult to make than balls)

- May be integral or inserts. Inserts should be potted with thin layer of epoxy

Materials

- Ceramics = low friction, high stiffness, and small contact points
- If using metals, harden
- Use dissimilar materials for ball and groove

Preparation and Assembly

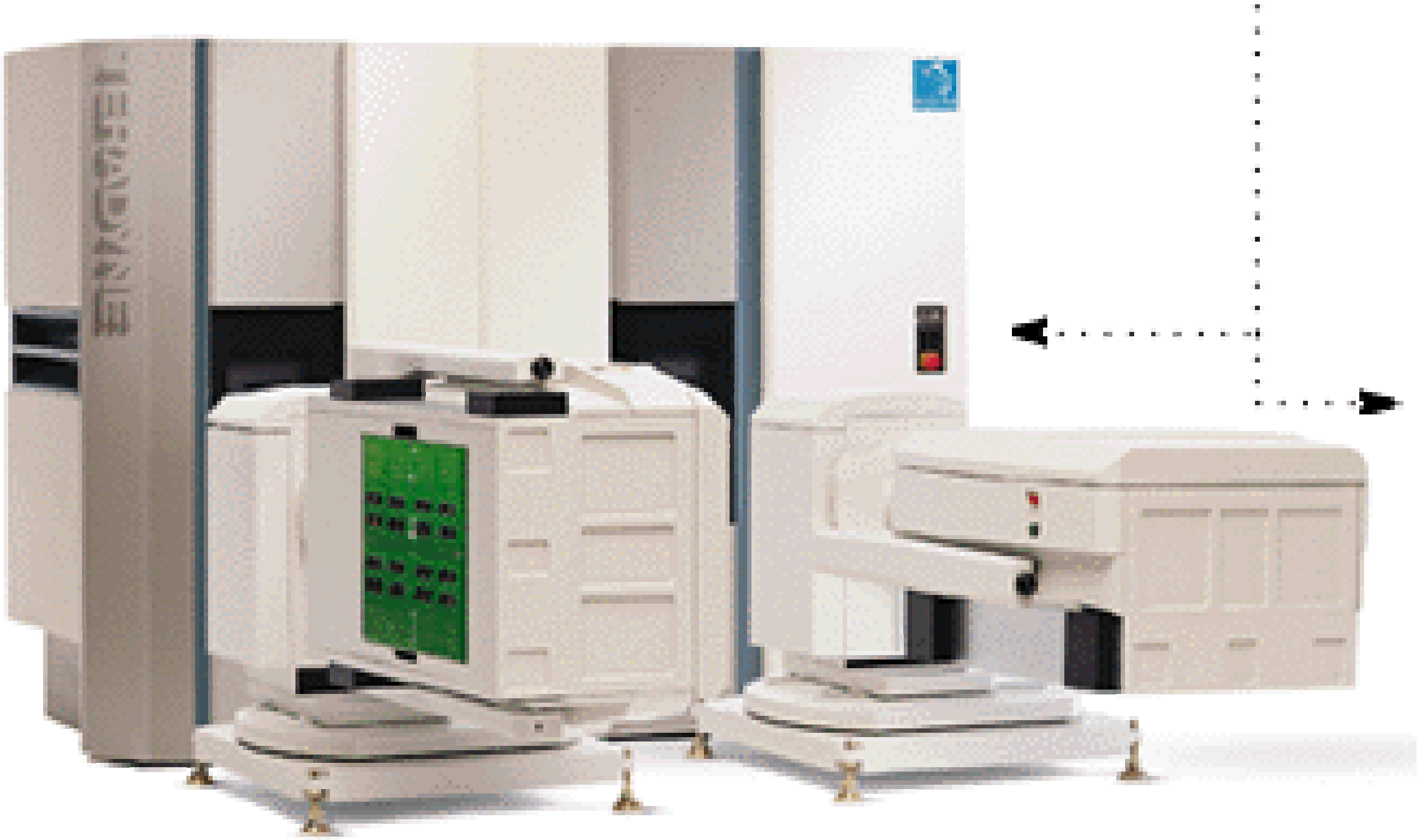
- Clean with oil mist
- Lubricate grooves if needed

Example: Servo-controlled kinematic couplings

Location & automatic leveling of precision electronic test equipment

Teradyne has shipped over 500 systems

Ph.D. Thesis: Michael Chiu



Example: Canoe-Ball kinematic interface element

The "Canoe Ball" shape is the secret to a highly repeatable design

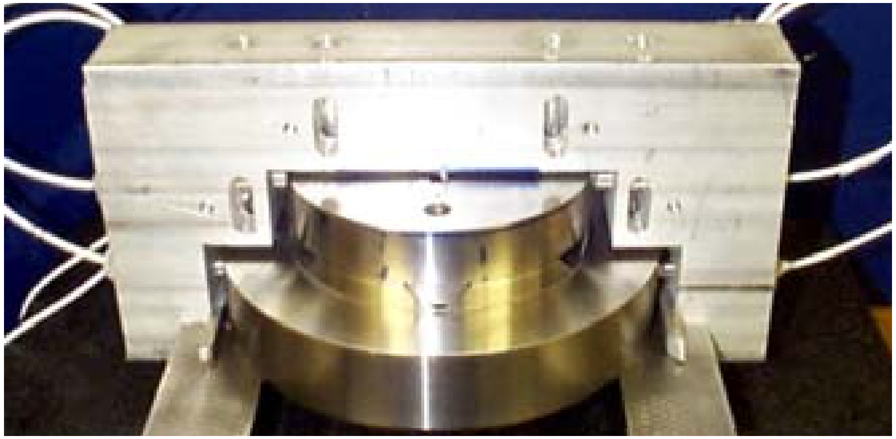
- **It acts like a ball 1 meter in diameter**
- **It has 100 times the stiffness and load capacity of a *normal* 1" ball**

Large, shallow Hertzian zone is *very (i.e. < 0.1 microns)* repeatable

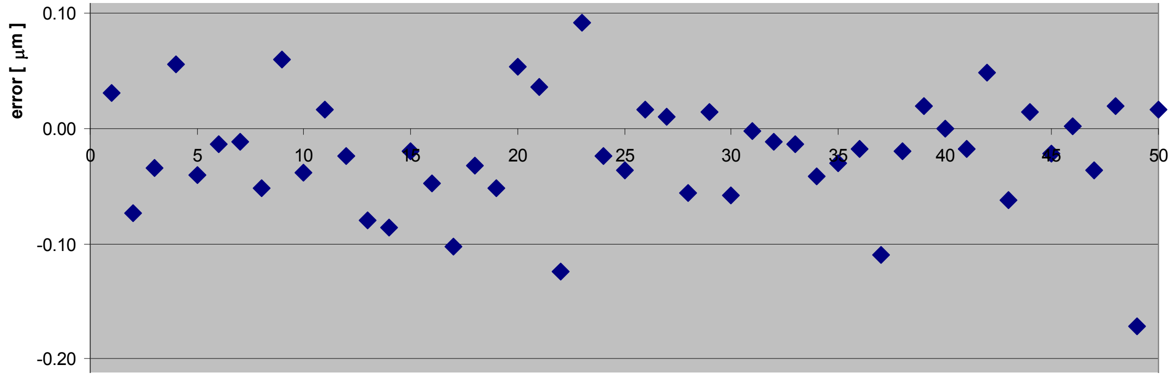
M.S. Thesis, Bernhard Muellerheld

Canoe-Ball repeatability measurements

Test Setup



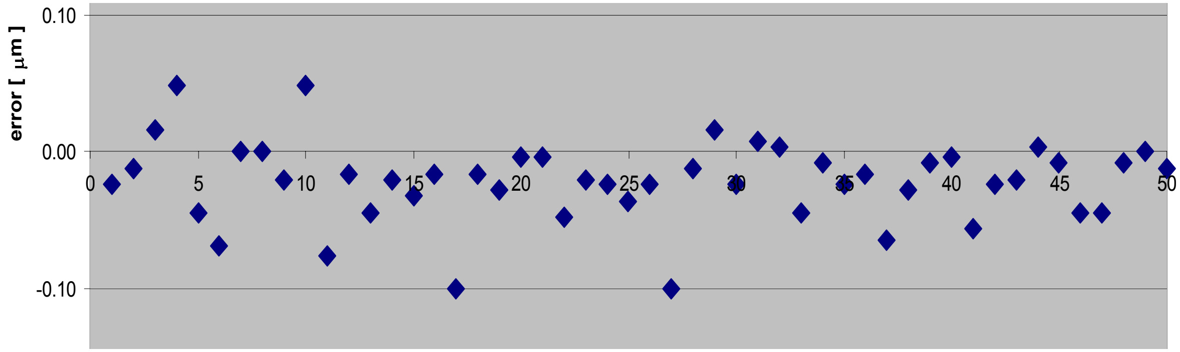
Coupling



0.1 µm

-0.2 µm

Meas. system



0.1 µm

-0.1 µm

There are MANY uses for Kinematic Couplings….

US005915678A

United States Patent [19]
Slocum et al.

[11] **Patent Number: 5,915,678**
[45] **Date of Patent: Jun. 29, 1999**

[54] **KINEMATIC COUPLING SYSTEM FOR THIN PLATES AND SHEETS AND THE LIKE**

[75] Inventors: **Alexander H. Slocum**, Bow, N.H.; **Daniel Braunstein**, Somerville, Mass.

[73] Assignee: **Aesop, Inc.**, Concord, N.H.

[21] Appl. No.: **08/655,456**

[22] Filed: **May 30, 1996**

[51] **Int. Cl.**[6] **B23Q 1/00**
[52] **U.S. Cl.** **269/47**; 269/305; 269/903; 29/281.1
[58] **Field of Search** 29/281.5, 281.1; 269/47, 50, 51, 52, 76, 903, 900, 296–298, 305; 211/41, 87, 14; 248/488–490

[56] **References Cited**

U.S. PATENT DOCUMENTS

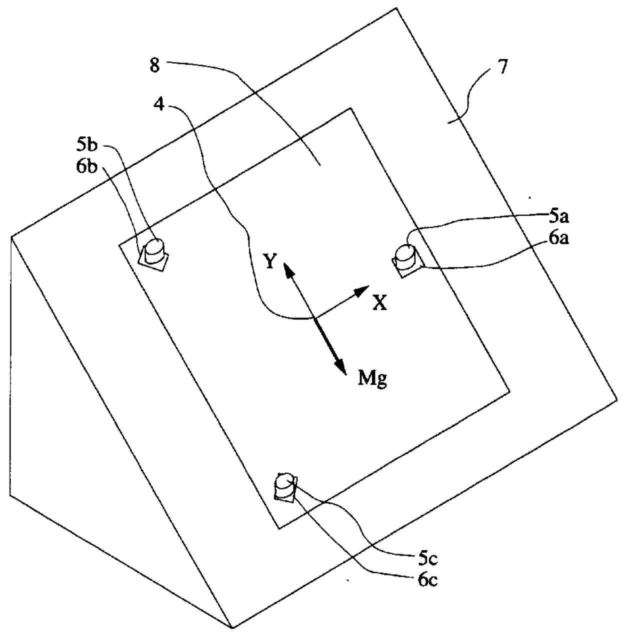
2,995,357	8/1961	Dennis et al.	269/51
4,875,966	10/1989	Perko	269/903
5,566,840	10/1996	Waldner et al.	269/47

Primary Examiner—Robert C. Watson
Attorney, Agent, or Firm—Rines and Rines

[57] **ABSTRACT**

A novel system for kinematically positioning planar surfaces such as printed circuit board components and elements of material, where one element has precisely formed and positioned (e.g., punched) slots that fit loosely over three corresponding posts, such that when a bias force, such as gravity from a tilted plane or a combing action, or expanding bladder on the posts, is applied, the surfaces are referenced to one side of the slots to in effect form a planar kinematic coupling so the relative planar position and orientation is defined with great accuracy and repeatability; such also allowing an entire factory system to be built where each piece of precision manufacturing equipment can use this planar kinematic coupling to achieve great accuracy in the manufacture of printed circuit boards or the like to allow for the use of ever smaller electronics components.

6 Claims, 4 Drawing Sheets



The Kinematic Sheet Coupling was created for the PCB industry

It provides 10x greater repeatability than traditional 4 pins-in-4-slots method