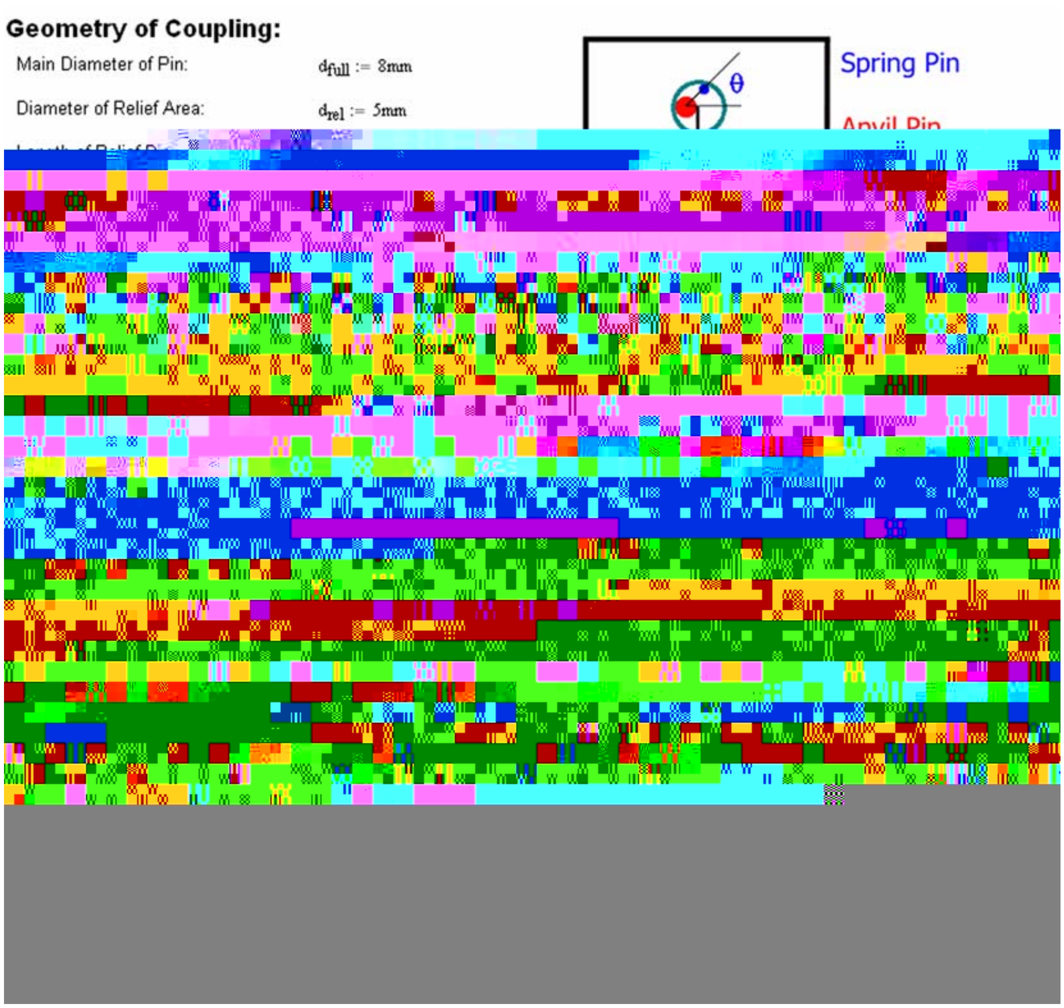
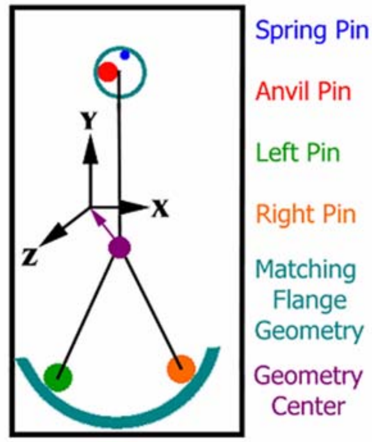
# Appendix B

## MATHEMATICS FOR THREE PIN COUPLING



**Applied Forces and Moments:**

| | |
|---|---|
| Force in X Direction (N): | $F_x := 0N$ |
| Force in Y Direction (N): | $F_y := 15N$ |
| Force in Z Direction (N): | $F_z := 0N$ |
| Moment in X Direction (N-m): | $M_x := 0N \cdot m$ |
| Moment in Y Direction (N-m): | $M_y := 0N \cdot m$ |
| Moment in Z Direction (N-m): | $M_z := 0N \cdot m$ |
| X Location of Forces (mm): | $x := 0mm$ |
| Y Location of Forces (mm): | $y := 0mm$ |
| Z Location of Forces (mm): | $z := 0mm$ |
| X Rotation of Forces (deg): | $x_{rot} := 0deg$ |
| Y Rotation of Forces (deg): | $y_{rot} := 0deg$ |
| Z Rotation of Forces (deg): | $z_{rot} := 0deg$ |



End Displacement of Pin:

$$y := \frac{d_{full} - d_{rel}}{2} \qquad y = 1.5\,mm$$

Moments of Inertia:

$$r_{full} := \frac{d_{full}}{2} \qquad I_{full} := \frac{1}{2}\cdot\pi\cdot r_{full}^{4} \qquad I_{full} = 402.124\,mm^{4}$$

$$r_{rel} := \frac{d_{rel}}{2} \qquad I_{rel} := \frac{1}{2}\cdot\pi\cdot r_{rel}^{4} \qquad I_{rel} = 61.359\,mm^{4}$$

Additional Lengths:

Total External Pin Length: $L_{pin} := L_{rel} + L_{force}$ $\qquad L_{pin} = 80\,mm$

Length to Force Location on Pin: $L_f := L_{rel} + \frac{L_{force}}{2}$ $\qquad L_f = 70\,mm$

Force Acting at End of Pin like a Simple Cantilever Beam for Deflection of y:

Forces for Deflections on Pin with Full Diameter:

$$F_{full} := \frac{y\cdot 3\cdot E\cdot I_{full}}{L_{pin}^{3}} \qquad F_{full} = 247.4\,N$$

Forces for Deflections on Pin with Relief Diameter:

$$F_{rel} := \frac{y\cdot 3\cdot E\cdot I_{rel}}{L_{pin}^{3}} \qquad F_{rel} = 37.75\,N$$

Bending Moment and Stress at Base of Pin:

$$M := F_{rel}\cdot L_f \qquad M = 2.643\,N\cdot m$$

$$\sigma := \frac{M\cdot r_{rel}}{I_{rel}} \qquad \sigma = 107.666\,10^{6}\,Pa$$

If more complex pins are used (ie. changing cross section along pin length), then force and deflections must be adjusted to reflect the change. This analysis assumes that the reliefe diameter will bend significantly more than the full diameter at the pin head. Some minor additional deflection will occur along the length of the pin head, but this amount should be less than along the relief. Complete force will be slightly larger than Frel, but significantly smaller than Ffull.

In addition, if spring pin is not used, Frel can be specified to analyze the force summation of a generic three pin coupling.

Determination of total in plane load caused by spring pin preload and frictional resistance:

$$F_{plane} := F_{rel} - \mu \cdot F_z$$

Position Matrix from Summation of Forces and Moments using Free Body Diagram:

$$A := \begin{bmatrix} 1m & \sin(\alpha)m & -\sin(\beta)m & 0m & 0m & 0m \\ 0m & \cos(\alpha)m & \cos(\beta)m & 0m & 0m & 0m \\ 0m & 0m & 0m & 1m & 1m & 1m \\ 0m & z\cdot\cos(\alpha) & z\cdot\cos(\beta) & -(L_{cp}-y) & y + r_L\cdot\cos(\alpha) & y + r_R\cdot\cos(\beta) \\ -z & -z\cdot\sin(\alpha) & z\cdot\sin(\beta) & -x & -(x + r_L\cdot\sin(\alpha)) & -(x - r_R\cdot\sin(\beta)) \\ y - L_{cp} & -x\cdot\cos(\alpha) + y\cdot\sin(\alpha) & -(x\cdot\cos(\beta) + y\cdot\sin(\beta)) & 0m & 0m & 0m \end{bmatrix} \quad A = \begin{pmatrix} 1 & 0.5 & -0.5 & 0 & 0 & 0 \\ 0 & 0.866 & 0.866 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & -0.061 & 0.036 & 0.036 \\ 0 & 0 & 0 & 0 & -0.02 & 0.02 \\ -0.061 & 7.5\times10^{-4} & -7.5\times10^{-4} & 0 & 0 & 0 \end{pmatrix} m$$

Force Matrix from Summation of Forces and Moments using Free Body Diagram:

$$V := \begin{bmatrix} \left[-\left[\cos(z_{rot})\cdot\cos(y_{rot})\cdot F_x + (\cos(z_{rot})\cdot\sin(y_{rot})\cdot\sin(x_{rot}) - \sin(z_{rot})\cdot\cos(x_{rot}))\cdot F_y + (\cos(z_{rot})\cdot\sin(y_{rot})\cdot\cos(x_{rot}) + \sin(z_{rot})\cdot\sin(x_{rot}))\cdot F_z\right] + F_{plane}\cdot\cos(\theta)\right]m \\ \left[-\left[\sin(z_{rot})\cdot\cos(y_{rot})\cdot F_x + (\sin(z_{rot})\cdot\sin(y_{rot})\cdot\sin(x_{rot}) + \cos(z_{rot})\cdot\cos(x_{rot}))\cdot F_y + (\sin(z_{rot})\cdot\sin(y_{rot})\cdot\cos(x_{rot}) - \cos(z_{rot})\cdot\sin(x_{rot}))\cdot F_z\right] + F_{plane}\cdot\sin(\theta)\right]m \\ (-\sin(y_{rot})\cdot F_x + \cos(y_{rot})\cdot\sin(x_{rot})\cdot F_y + \cos(y_{rot})\cdot\cos(x_{rot})\cdot F_z)m \\ -\left[\cos(z_{rot})\cdot\cos(y_{rot})\cdot M_x + (\cos(z_{rot})\cdot\sin(y_{rot})\cdot\sin(x_{rot}) - \sin(z_{rot})\cdot\cos(x_{rot}))\cdot M_y + (\cos(z_{rot})\cdot\sin(y_{rot})\cdot\cos(x_{rot}) + \sin(z_{rot})\cdot\sin(x_{rot}))\cdot M_z\right] + z\cdot\sin(\theta)\cdot F_{plane} \\ -\left[\sin(z_{rot})\cdot\cos(y_{rot})\cdot M_x + (\sin(z_{rot})\cdot\sin(y_{rot})\cdot\sin(x_{rot}) + \cos(z_{rot})\cdot\cos(x_{rot}))\cdot M_y + (\sin(z_{rot})\cdot\sin(y_{rot})\cdot\cos(x_{rot}) - \cos(z_{rot})\cdot\sin(x_{rot}))\cdot M_z\right] - z\cdot\cos(\theta)\cdot F_{plane} \\ -(-\sin(y_{rot})\cdot M_x + \cos(y_{rot})\cdot\sin(x_{rot})\cdot M_y + \cos(y_{rot})\cdot\cos(x_{rot})\cdot M_z) - (x\cdot\sin(\theta) - y\cdot\cos(\theta) + L_{cp}\cdot\cos(\theta))\cdot F_{plane} \end{bmatrix}$$

$$V = \begin{pmatrix} 32.693 \\ 3.875 \\ 0 \\ 0 \\ 0 \\ -1.994 \end{pmatrix} N\cdot m$$

Solving for forces applied at contact points: $F := \text{lsolve}(A, V)$

Which gives the following forces:

| | | | |
|---|---|---|---|
| $F_{anvil} := F_{0,0}$ | $F_{anvil} = 32.693\,N$ | $F_{z\_bolt\_at\_control\_pin} := F_{3,0}$ | $F_{z\_bolt\_at\_control\_pin} = 0\,N$ |
| $F_{left} := F_{1,0}$ | $F_{left} = 2.237\,N$ | $F_{z\_bolt\_at\_left\_pin} := F_{4,0}$ | $F_{z\_bolt\_at\_left\_pin} = 0\,N$ |
| $F_{right} := F_{2,0}$ | $F_{right} = 2.237\,N$ | $F_{z\_bolt\_at\_right\_pin} := F_{5,0}$ | $F_{z\_bolt\_at\_right\_pin} = 0\,N$ |