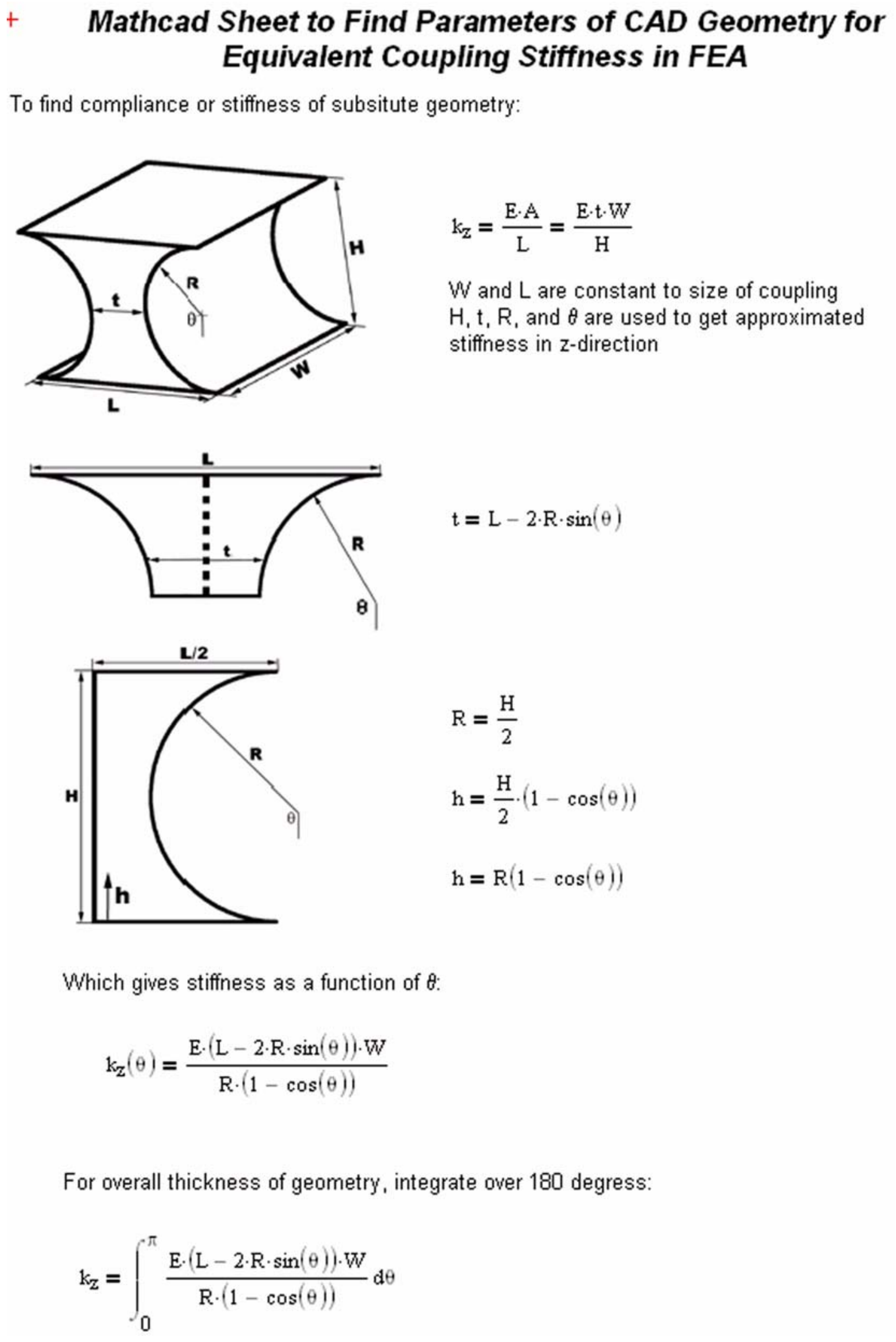
Appendix C

STIFFNESS APPROXIMATIONS FOR FEA OF COUPLING SIMULATION



Mathcad Sheet to Find Parameters of CAD Geometry for Equivalent Coupling Stiffness in FEA

To find compliance or stiffness of subsitute geometry:

$$k_z = \frac{E\cdot A}{L} = \frac{E\cdot t\cdot W}{H}$$

W and L are constant to size of coupling
H, t, R, and *θ* are used to get approximated stiffness in z-direction

$$t = L - 2\cdot R\cdot \sin(\theta)$$

$$R = \frac{H}{2}$$

$$h = \frac{H}{2}\cdot(1 - \cos(\theta))$$

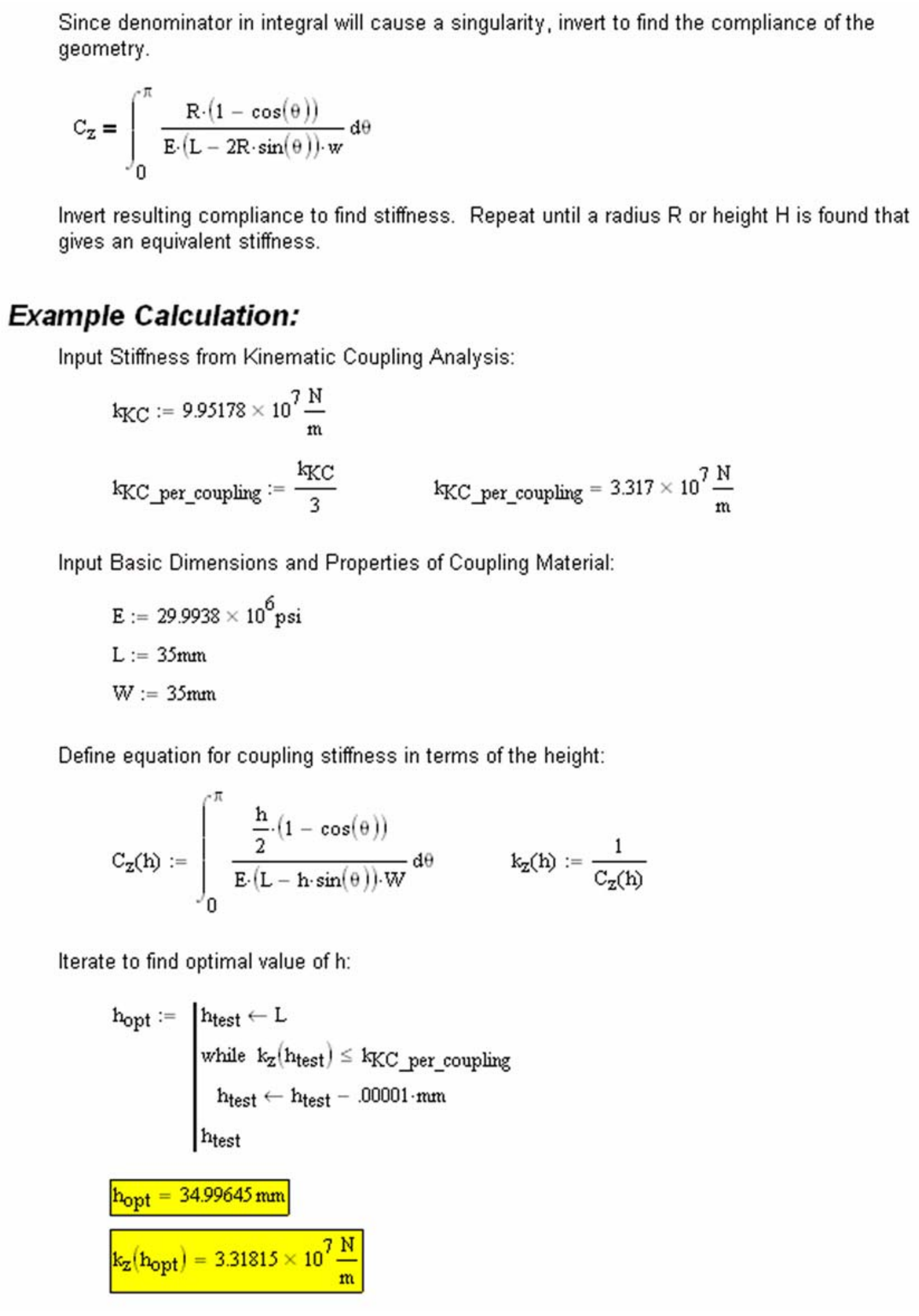
$$h = R(1 - \cos(\theta))$$

Which gives stiffness as a function of *θ*:

$$k_z(\theta) = \frac{E\cdot(L - 2\cdot R\cdot \sin(\theta))\cdot W}{R\cdot(1 - \cos(\theta))}$$

For overall thickness of geometry, integrate over 180 degress:

$$k_z = \int_0^{\pi} \frac{E\cdot(L - 2\cdot R\cdot \sin(\theta))\cdot W}{R\cdot(1 - \cos(\theta))}\,d\theta$$



Since denominator in integral will cause a singularity, invert to find the compliance of the geometry.

$$C_z = \int_0^{\pi} \frac{R\cdot(1-\cos(\theta))}{E\cdot(L-2R\cdot\sin(\theta))\cdot w}\,d\theta$$

Invert resulting compliance to find stiffness. Repeat until a radius R or height H is found that gives an equivalent stiffness.

Example Calculation:

Input Stiffness from Kinematic Coupling Analysis:

$$k_{KC} := 9.95178 \times 10^{7}\,\frac{N}{m}$$

$$k_{KC\_per\_coupling} := \frac{k_{KC}}{3} \qquad k_{KC\_per\_coupling} = 3.317 \times 10^{7}\,\frac{N}{m}$$

Input Basic Dimensions and Properties of Coupling Material:

$$E := 29.9938 \times 10^{6}\,psi$$

$$L := 35mm$$

$$W := 35mm$$

Define equation for coupling stiffness in terms of the height:

$$C_z(h) := \int_0^{\pi} \frac{\frac{h}{2}\cdot(1-\cos(\theta))}{E\cdot(L-h\cdot\sin(\theta))\cdot W}\,d\theta \qquad k_z(h) := \frac{1}{C_z(h)}$$

Iterate to find optimal value of h:

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$h_{opt} :=$ | $h_{test} \leftarrow L$
             | while $k_z(h_{test}) \le k_{KC\_per\_coupling}$
             |     $h_{test} \leftarrow h_{test} - .00001\cdot mm$
             | $h_{test}$
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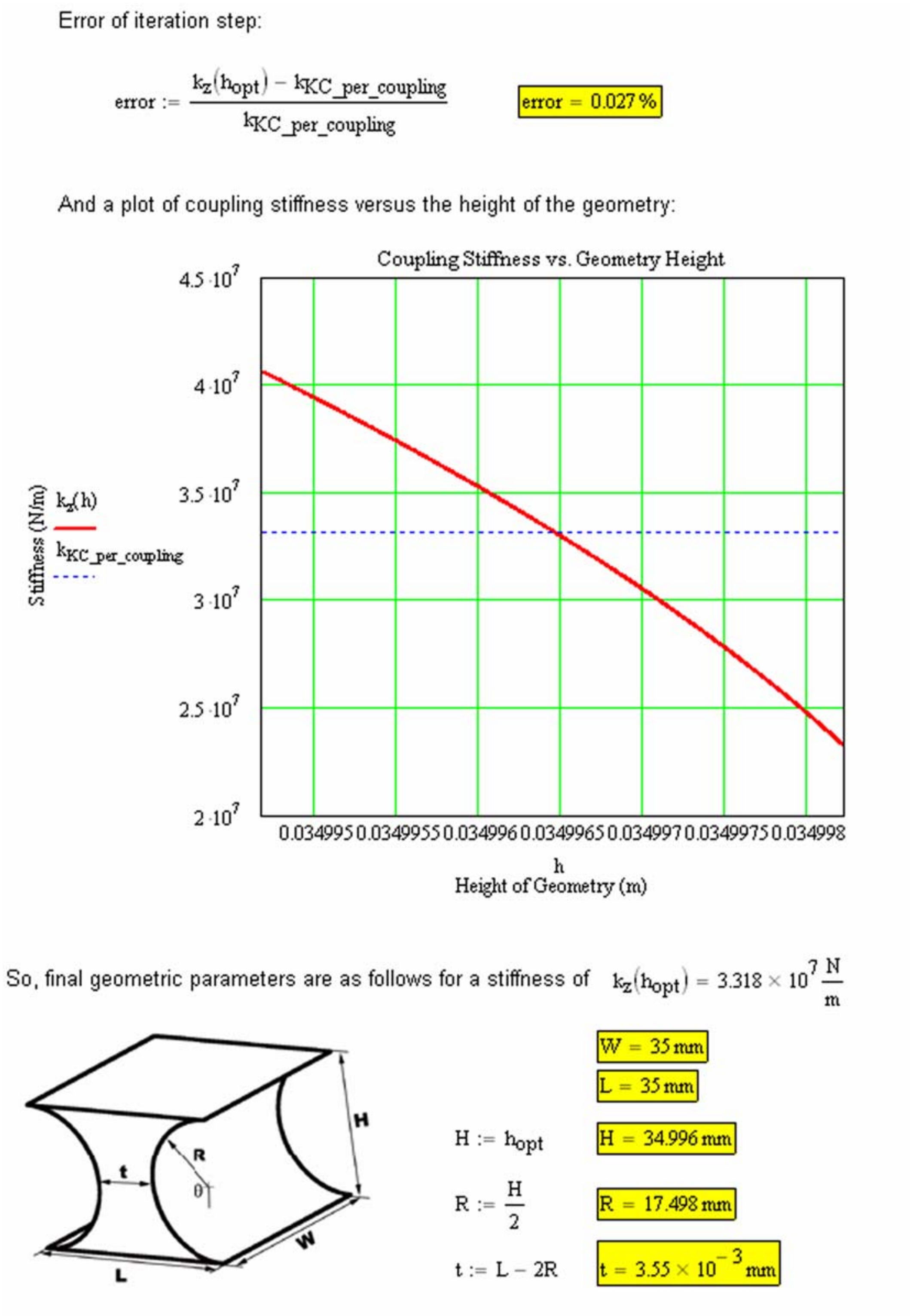
$$h_{opt} = 34.99645\,mm$$

$$k_z(h_{opt}) = 3.31815 \times 10^{7}\,\frac{N}{m}$$



Error of iteration step:

$$\text{error} := \frac{k_z(h_{opt}) - k_{KC\_per\_coupling}}{k_{KC\_per\_coupling}} \qquad \text{error} = 0.027\,\%$$

And a plot of coupling stiffness versus the height of the geometry:

Coupling Stiffness vs. Geometry Height

So, final geometric parameters are as follows for a stiffness of $k_z(h_{opt}) = 3.318 \times 10^7 \frac{N}{m}$

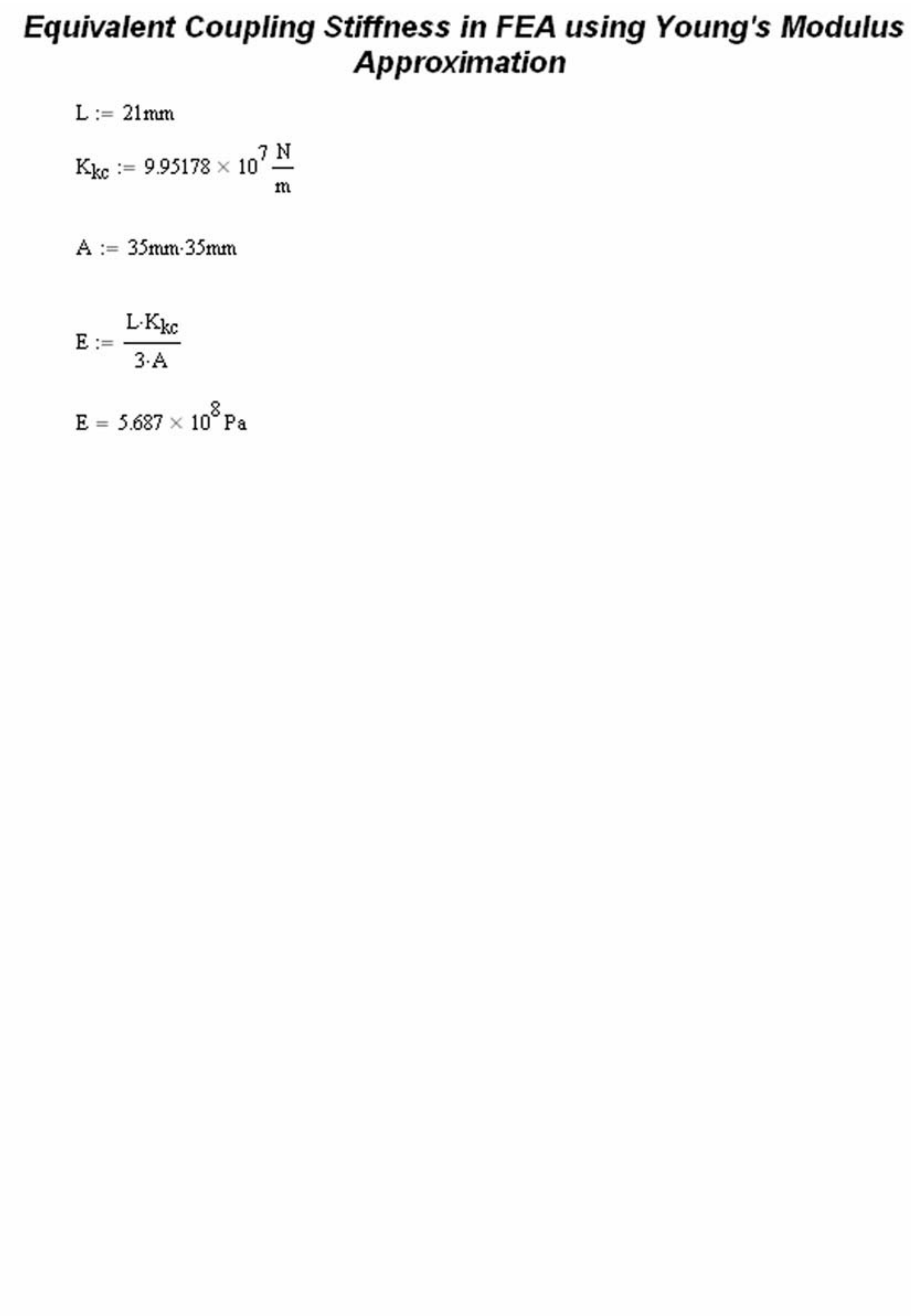
$W = 35\,mm$

$L = 35\,mm$

$H := h_{opt}$ $\quad H = 34.996\,mm$

$R := \frac{H}{2}$ $\quad R = 17.498\,mm$

$t := L - 2R$ $\quad t = 3.55 \times 10^{-3}\,mm$



Equivalent Coupling Stiffness in FEA using Young's Modulus Approximation

$L := 21\text{mm}$

$K_{kc} := 9.95178 \times 10^{7} \frac{N}{m}$

$A := 35\text{mm} \cdot 35\text{mm}$

$$E := \frac{L \cdot K_{kc}}{3 \cdot A}$$

$E = 5.687 \times 10^{8} \text{Pa}$