Appendix C

STIFFNESS APPROXIMATIONS FOR FEA OF COUPLING SIMULATION
**Mathcad Sheet to Find Parameters of CAD Geometry for Equivalent Coupling Stiffness in FEA**

To find compliance or stiffness of substitute geometry:

\[ k_z = \frac{E \cdot A}{L} = \frac{E \cdot t \cdot W}{H} \]

*W* and *L* are constant to size of coupling.

*H*, *t*, *R*, and *θ* are used to get approximated stiffness in z-direction.

\[ t = L - 2 \cdot R \cdot \sin(\theta) \]

\[ R = \frac{H}{2} \]

\[ h = \frac{H}{2} \cdot (1 - \cos(\theta)) \]

\[ h = R \cdot (1 - \cos(\theta)) \]

Which gives stiffness as a function of *θ*:

\[ k_z(\theta) = \frac{E \cdot (L - 2 \cdot R \cdot \sin(\theta)) \cdot W}{R \cdot (1 - \cos(\theta))} \]

For overall thickness of geometry, integrate over 180 degrees:

\[ k_z = \int_{0}^{\pi} \frac{E \cdot (L - 2 \cdot R \cdot \sin(\theta)) \cdot W}{R \cdot (1 - \cos(\theta))} \, d\theta \]
Since denominator in integral will cause a singularity, invert to find the compliance of the geometry.

\[ C_Z = \int_0^{\pi} \frac{R \cdot (1 - \cos(\theta))}{E \cdot (L - 2R \cdot \sin(\theta)) \cdot W} \, d\theta \]

Invert resulting compliance to find stiffness. Repeat until a radius \( R \) or height \( H \) is found that gives an equivalent stiffness.

**Example Calculation:**

Input Stiffness from Kinematic Coupling Analysis:

\[ k_{KC} := 9.95178 \times 10^7 \frac{N}{m} \]

\[ k_{KC\_per\_coupling} := \frac{k_{KC}}{3} \quad k_{KC\_per\_coupling} = 3.317 \times 10^7 \frac{N}{m} \]

Input Basic Dimensions and Properties of Coupling Material:

\[ E := 29.9938 \times 10^6 \text{ psi} \]
\[ L := 35\text{mm} \]
\[ W := 35\text{mm} \]

Define equation for coupling stiffness in terms of the height:

\[ C_Z(h) := \int_0^{\pi} \frac{h}{2} \frac{(1 - \cos(\theta))}{E \cdot (L - h \cdot \sin(\theta)) \cdot W} \, d\theta \]

\[ k_2(h) := \frac{1}{C_2(h)} \]

Iterate to find optimal value of \( h \):

\[ h_{opt} := \begin{align*}
h_{test} & \leftarrow L \\
\text{while } & k_2(h_{test}) \leq k_{KC\_per\_coupling} \\
& h_{test} \leftarrow h_{test} - .00001 \text{mm} \\
h_{test} & \end{align*} \]

\[ h_{opt} = 34.99645 \text{mm} \]

\[ k_2(h_{opt}) = 3.31815 \times 10^7 \frac{N}{m} \]
Error of iteration step:

\[ \text{error} := \frac{k_c(h_{opt}) - k_{KC\_per\_coupling}}{k_{KC\_per\_coupling}} \]

And a plot of coupling stiffness versus the height of the geometry:

![Coupling Stiffness vs. Geometry Height](image)

So, final geometric parameters are as follows for a stiffness of \( k_c(h_{opt}) = 3.318 \times 10^7 \frac{N}{m} \):

- \( W = 35 \text{ mm} \)
- \( L = 35 \text{ mm} \)
- \( H := h_{opt} \) \( H = 34.996 \text{ mm} \)
- \( R := \frac{H}{2} \) \( R = 17.498 \text{ mm} \)
- \( t := L - 2R \) \( t = 3.55 \times 10^{-3} \text{ mm} \)
Equivalent Coupling Stiffness in FEA using Young's Modulus Approximation

\[ L := 21 \text{ mm} \]

\[ K_{kc} := 9.95178 \times 10^7 \frac{N}{m} \]

\[ A := 35\text{mm}-35\text{mm} \]

\[ E := \frac{L \cdot K_{kc}}{3 \cdot A} \]

\[ E = 5.687 \times 10^8 \text{ Pa} \]